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Grounding Resistance Calculation of Plate Board Electrode Using Reduction Materials

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Abstract **Original Research Article**

In this paper, a mathematical model on the grounding resistance of the plate board electrode by using reduction material is proposed. The proposed mathematical model is modeled with consideration of the effects of parameters such as the resistivity and thickness of reduction material and embedding depth.

This mathematical model can be applied effectively to the design of grounding system using reduction material in harmonious soil with the resistivity values between and.

The accuracy of proposed model is proved by the simulation analysis and measurement examination of the electric field in this paper.

Keywords: plate board electrode, reduction material, grounding resistance, embedding depth.

Introduction

The technical requirement on the earthing and lightning system protection becomes strictly more and more in the right of the increasing public sensitivity with regard to power available in electrical power systems. In Electrical safety, earthing and lightning system protection is a mechanism that is employed to dispose undesirable electrical charge to the earth mass or send back the electrical charge to the generator via earth mass (Ameb, Onuehbu & Nwababbe, 2022, Javanovic & Vcetkovic, 2016).

The availability of a grounding system must have the smallest grounding resistance value (Kalyani & A. K, 2013). Low grounding resistance at individual structures improves safety and reduces back flashover stress from lightning surge currents (Su Mon & kgin Thidar & Thient Tun 2019). Therefore, the grounding resistance is an important parameter in the appraisal on the effectiveness and safety of the grounding system (Tingfang & Wei Qiu, 2015).

The mathematical models on the grounding resistance of the plate board electrode have researched in (Vasilios & Ioannis, Gonos & Stathopulos, 2015). However, in the literatures, the resistivity around the electrode has been considered to be constant for the mathematical model on the grounding resistance of the plate board electrode and the influence of the embedding depth is ignored.

The aim of the present work is to get the mathematical model considering the influence of the resistivity and thickness of the reduction material and embedding depth synthetically in consideration of using reduction material and to confirm the propriety of the model by the results of the computer simulation and field measurement examination in the condition of different soil resistivity.

The paper consists of 4sections. In section 2, the grounding resistances of the plate board electrode with and without reduction material analyzed theoretically. Section 3 gives the results and the Section 4 concludes this paper.

II. Theoretical analysis on grounding resistance of plate board electrode

- **Considering of no reduction material**

The plate board electrode buried in homogeneous soil is shown in Fig. 1.

Where, L is the length of one side of plate board electrode and h is embedding depth.

The thickness of plate is ignored.

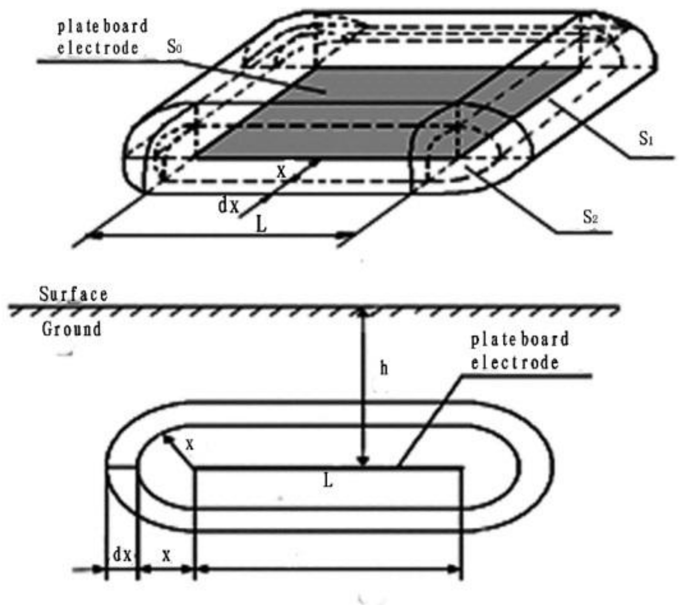

Figure1.plate board electrode that is buried in homogeneous soil

½ body   2) a cross section

1)

Considering of the element volume with dx of thickness at the point which is away x from plate.

- $0 < x \leq h$

In this area, the resistance of the element volume dR can be written:

$$dR = \frac{\rho}{2S_0 + 8S_1 + 8S_2} dx \tag{1}$$

where,

$$S_0 = L^2 \quad S_2 = \frac{\pi}{2}x^2 \quad S_1 = \frac{\pi}{2}xL$$

Substitute these into equation (1), the result is

$$dR = \frac{\rho}{4\pi x^2 + 4\pi Lx + 2L^2}dx \quad (2)$$

So the resistance of this area is

$$R'(x) = \int_0^h \frac{\rho}{4\pi x^2 + 4\pi Lx + 2L^2}dx \quad (3)$$

- h < x < ∞

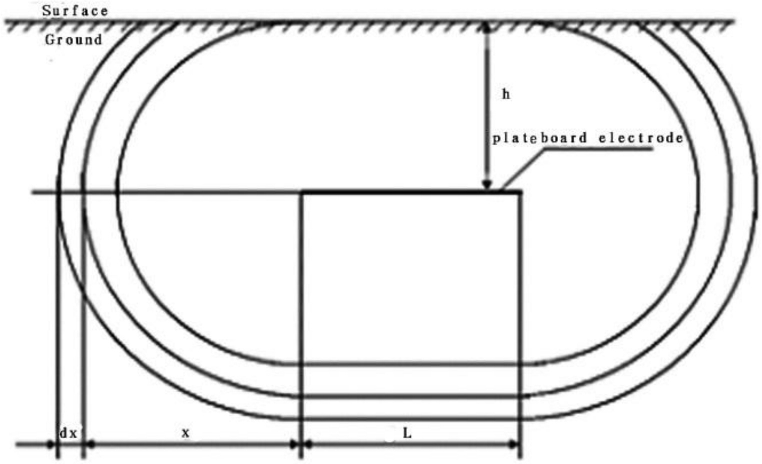

Figure2. In the case of h<x, equipotential distribution of the plate board electrode

In this area, the resistance of the space element dR can be written as following.

$$dR = \frac{\rho}{S_0 + 4S_1 + 4S_2 + S_3 + S_4}dx \quad (4)$$

where,

$$S_3 = 2\pi xh, \qquad S_4 = 4Lx\arctan\frac{h}{\sqrt{x^2 - h^2}}$$

-Case of x ≫ h $\quad \arctan\frac{h}{\sqrt{x^2-h^2}} \approx \frac{h}{x}$

So equation (4) can be written as following.

$$S_4 = 4Lx \cdot \frac{h}{x} = 4Lh$$

Substitute it into equation 4, the result is

$$R''(x) = \int_h^\infty \frac{\rho}{2\pi x^2 + 2\pi(L+h)x + L^2 + 4Lh}dx \quad (5)$$

So a total resistance is as following.

$$R(x) = R'(x) + R''(x) = \int_0^h \frac{\rho}{4\pi x^2+4\pi Lx+2L^2}dx + \int_h^\infty \frac{\rho}{2\pi x^2+2\pi(L+h)x+L^2+4Lh}dx \quad (6)$$

Solving this equation (6),

$$R(x) = \frac{\rho}{\sqrt{b_1^2 - 4a_1c_1}} \ln \frac{h\left(b_1 + \sqrt{b_1^2 - 4a_1c_1}\right) + 2c_1}{h\left(b_1 - \sqrt{b_1^2 - 4a_1c_1}\right) + 2c_1} + \frac{\rho}{\sqrt{b_2^2 - 4a_2c_2}} \ln \frac{2a_2h + b_2 + \sqrt{b_2^2 - 4a_2c_2}}{2a_2h + b_2 - \sqrt{b_2^2 - 4a_2c_2}} \quad (7)$$

where,

$a_1 = 4\pi,\ b_1 = 4\pi L,\ c_1 = 2L^2$

$a_2 = 2\pi,\ b_2 = 2\pi(L + h),\ c_2 = L^2 + 4Lh$

Considering of coefficient, equation 7 can be written as following.

$$R(x) = \frac{\rho}{\sqrt{b_1^2 - 4a_1c_1}} \ln k_1 \cdot \frac{h\left(b_1 + \sqrt{b_1^2 - 4a_1c_1}\right) + 2c_1}{h\left(b_1 - \sqrt{b_1^2 - 4a_1c_1}\right) + 2c_1} + \frac{\rho}{\sqrt{b_2^2 - 4a_2c_2}} \ln k_1 \cdot \frac{2a_2h + b_2 + \sqrt{b_2^2 - 4a_2c_2}}{2a_2h + b_2 - \sqrt{b_2^2 - 4a_2c_2}} \quad (8)$$

where, $k_1 = 1.13$.

- Considering of reduction material

In consideration of reduction material, grounding resistance equation is as following.

$$R(x) = R_0(x) - R_{01}(x) + R_{02}(x) = R_0(x) - \Delta R(x)$$

Where,

$R_0(x)$- grounding resistance before using reduction material.

$R_{01}(x)$- resistance of main soil according to area that has reduction material.

$R_{02}(x)$- resistance according to area that has reduction material.

Where, t is the thickness of reduction material.

$$R_{01}(x) = \int_0^t \frac{\rho}{4\pi x^2 + 4\pi Lx + 2L^2} dx = \frac{\rho}{\sqrt{b_1^2 - 4a_1c_1}} \ln \frac{t\left(b_1 + \sqrt{b_1^2 - 4a_1c_1}\right) + 2c_1}{t\left(b_1 - \sqrt{b_1^2 - 4a_1c_1}\right) + 2c_1} \quad (9)$$

$$R_{02}(x) = \int_0^t \frac{\rho_c}{4\pi x^2 + 4\pi Lx + 2L^2} dx = \frac{\rho_c}{\sqrt{b_1^2 - 4a_1c_1}} \ln \frac{t\left(b_1 + \sqrt{b_1^2 - 4a_1c_1}\right) + 2c_1}{t\left(b_1 - \sqrt{b_1^2 - 4a_1c_1}\right) + 2c_1} \quad (10)$$

$$\Delta R(x) = R_{01}(x) - R_{02}(x) = \frac{\rho - \rho_c}{\sqrt{b_1^2 - 4a_1c_1}} \ln \frac{t\left(b_1 + \sqrt{b_1^2 - 4a_1c_1}\right) + 2c_1}{t\left(b_1 - \sqrt{b_1^2 - 4a_1c_1}\right) + 2c_1} \quad (11)$$

Considering of coefficient, equation (11) can be written:

$$\Delta R(x) = \frac{\rho - \rho_c}{\sqrt{b_1^2 - 4a_1c_1}} \ln \frac{k_2t\left(b_1 + \sqrt{b_1^2 - 4a_1c_1}\right) + 2c_1}{k_2t\left(b_1 - \sqrt{b_1^2 - 4a_1c_1}\right) + 2c_1} \quad (12)$$

where, $k_2 = 2$.

So total resistance is as following.

$$R(x)=R_0(x)-\Delta R(x)=\frac{\rho}{\sqrt{{b_1}^2-4a_1c_1}}\ln 1.13\cdot\frac{h\left(b_1+\sqrt{{b_1}^2-4a_1c_1}\right)+2c_1}{h\left(b_1-\sqrt{{b_1}^2-4a_1c_1}\right)+2c_1}+$$

$$+\frac{\rho}{\sqrt{{b_2}^2-4a_2c_2}}\ln 1.13\cdot\frac{2a_2h+b_2+\sqrt{{b_2}^2-4a_2c_2}}{2a_2h+b_2-\sqrt{{b_2}^2-4a_2c_2}}-\frac{\rho-\rho_c}{\sqrt{{b_1}^2-4a_1c_1}}\ln\frac{2t\left(b_1+\sqrt{{b_1}^2-4a_1c_1}\right)+2c_1}{2t\left(b_1-\sqrt{{b_1}^2-4a_1c_1}\right)+2c_1} \quad (13)$$

III. Investigation on the accuracy of the mathematical model by simulation analysis.

To confirm the precision of the mathematical model on the grounding resistance of plate board electrode using reduction material, ANSYS program is applied for simulation analysis. According to various thickness and resistivity of the reduction material and embedding depths of the plate board electrode, the grounding resistances is calculated by the Eq. (13) and are simulated by ANSYS program.

-Analysis and calculation condition.

Embedding depth $h = 0.8\text{m}$, soil resistivity $\rho = 800\Omega\text{m}$, resistivity of reduction material $\rho_c = 8\Omega\text{m}$.

Analysis and calculation results, deviation and deviation rate are shown in below table1, 2.

The thickness of reduction material is limited to about 40% of the length of one side of the plate.

Where,

R – the calculation value of grounding resistance by the Eq. (13), Ω.

R0 – the analysis value of grounding resistance by simulation examination, Ω..

Table 1 Calculation values and analysis values of grounding resistance $S = 1.0 \times 1.0\text{m}^2$

Thickness of reduction material d, m	Grounding resistance, Ω		deviation, Ω **ΔR**	deviation rate, % **ΔR**(%)
	R	R0		
0.02	166.39	170.94	4.55	2.66
0.10	130.93	134.87	3.94	2.92
0.20	107.46	109.73	2.27	2.07
0.40	83.99	82.95	-1.04	1.25

Table 2 Calculation values and analysis values of grounding resistance $S = 1.5 \times 1.5\text{m}^2$

Thickness of reduction material d, m	Grounding resistance, Ω		deviation, Ω **ΔR**	deviation rate, % **ΔR**(%)
	R	R0		
0.02	120.31	121.57	1.26	1.04
0.10	101.73	102.49	0.76	0.74

0.20	87.49	87.66	0.17	0.19
0.40	71.44	69.74	-1.7	2.44
0.50	66.42	64.13	2.29	3.57

In case of below condition, calculation values and analysis values of grounding resistance of the plate board electrode changing resistivity of reduction material differentially are shown in below table 3.

Area of plate board electrode $S = 1 \times 1m^2$, embedding depth $h = 2m$, soil resistivity $\rho = 800\Omega m$,

Resistivity of reduction material $\rho_c = 6\sim20\Omega m$, thickness of plate board electrode $d = 0.002m$, thickness of reduction material $t = 0.02m$.

Table 3 calculation values and analysis values of grounding resistance

Resistivity of reduction material $\rho_c, \Omega m$	Grounding resistance, Ω		deviation, Ω ΔR	deviation rate, % $\Delta R(\%)$
	R	R0		
6	166.35	170.87	4.52	2.64
10	166.42	171.00	4.58	2.68
14	166.49	171.14	4.65	2.71
18	166.56	171.27	4.71	2.75
20	166.60	171.33	4.73	2.76

Investigation about accuracy of mathematical model by field measurement examination.

The resistivity of ground is measured in the field by four poles method.

The measurement results are shown in table 4.

Table 4 Measurement values of soil resistivity

№	1	2	3	4	5	6
ρ, $\Omega \cdot m$	865	867	862	866	861	863

Calculating the arithmetical mean of the measurement values in table 4, the soil resistivity is $864\Omega \cdot m$. Also we made progress the field measurement examination on the grounding resistance of the plate board electrode changing the embedding depth, resistivity and thickness of reduction material differentially.

To confirm the accuracy of mathematical model on the grounding resistance of the plate board electrode using reduction material, the calculation value of grounding resistance by the Eq. (13) and calculation values of grounding resistance by the Eq. (8) in case of without considering reduction material are compared with the measurement values by the field measurement examination.

The measurement results are shown in table 5~10.

Embedding depth $h = 1m$, soil resistivity $\rho = 864\Omega m$, resistivity of reduction material $\rho_c = 8\Omega m$. Area of plate board electrode $S = 1 \times 1m^2$.

The measurement values according to the thickness of reduction material are shown in table5, 6, 7.

Where,

R1 – the calculation value of grounding resistance by the Eq. (13), Ω.

R2 – the calculation value of grounding resistance by the Eq. (8), Ω.

R0 – the analysis value of grounding resistance by simulation examination Ω.

Table 5 Calculation values by Eq. (13) and measurement values of grounding resistance

$\rho_c = 8\Omega m,\ \rho = 864\Omega \cdot m$

Thickness of reduction material d, m	Grounding resistance, Ω		deviation, Ω ΔR	deviation rate, % ΔR(%)
	R1	R0		
0.05	182.35	178.26	-4.09	2.24
0.20	136.22	134.02	-2.2	1.61
0.40	110.87	108.36	-2.51	2.26

Table 6 Calculation values by Eq. (13) and measurement values of grounding resistance

$\rho_c = 8\Omega m\ \rho = 864\Omega \cdot m$

Thickness of reduction material d, m	Grounding resistance, Ω		deviation, Ω ΔR	deviation rate, % ΔR(%)
	R1	R0		
0.05	182.35	179.72	-2.63	1.44
0.20	136.22	134.87	-1.35	0.99
0.40	110.87	108.78	-2.09	1.88

Table 7 Calculation values by Eq. (8) and measurement values of grounding resistance

$h = 1m,\ S = 1 \times 1m^2,\ \rho = 864\Omega \cdot m$

Thickness of reduction material d, m	Grounding resistance, Ω		deviation, Ω ΔR	deviation rate, % ΔR(%)
	R2	R0		
0.05	184.46	176.26	-8.2	4.65

0.20	184.46	132.05	-52.41	39.68
0.40	184.46	109.78	-74.68	68.02

Embedding depth $h = 1m$, soil resistivity $\rho = 864\Omega m$, area of plate board electrode $S = 1 \times 1㎡$.

The thickness of plate board $d = 0.002m$, the thickness of reduction material $t = 0.02m$.

The measurement values changing resistivity of reduction material differentially are shown in table8, 9.

Table 8 Calculation values by Eq. (13) and measurement values of grounding resistance

$t = 0.02m$, $\rho = 864\Omega \cdot m$

Resistivity of reduction material $\boldsymbol{\rho_c}$, **Ωm**	Grounding resistance, Ω		deviation, Ω **ΔR**	deviation rate, % **ΔR**(%)
	R1	R0		
8	199.91	203.78	3.87	1.93
20	200.13	204.97	4.84	2.41
40	200.48	205.14	4.66	2.32

Table 9 Calculation values by Eq. (8) and measurement values of grounding resistance

$h = 1m$, $S = 1 \times 1㎡$, $\rho = 864\Omega \cdot m$

Resistivity of reduction material $\boldsymbol{\rho_c}$, **Ωm**	Grounding resistance, Ω		deviation, Ω **ΔR**	deviation rate, % **ΔR**(%)
	R2	R0		
8	184.46	202.34	17.88	8.83
20	184.46	205.47	21.01	10.22
40	184.46	206.93	22.47	10.85

The field measurement examinations on the grounding resistance of plate board electrode and comparison analysis between the results and calculation values have been done when the resistivity of soil was $523\Omega \cdot m$.

The results are shown in below table 10-14.

Table 10 Calculation values by Eq. (13) and measurement values of grounding resistance

$\rho_c = 8\Omega m$ $\rho = 523\Omega \cdot m$

Thickness of reduction material d, m	Grounding resistance, Ω		deviation, Ω **ΔR**	deviation rate, % **ΔR**(%)
	R1	R0		
0.05	110.96	112.76	1.8	1.62
0.20	83.85	85.68	1.83	2.18
0.40	68.96	68.96	1.07	1.55

Table 11 Calculation values by Eq. (13) and measurement values of grounding resistance

$\rho_c = 8\Omega\mathrm{m}$ $\rho = 523\Omega \cdot m$

Thickness of reduction material d, m	Grounding resistance, Ω		deviation, Ω **ΔR**	deviation rate, % **ΔR**(%)
	R1	R0		
0.05	110.96	113.46	2.5	2.25
0.20	83.85	86.71	2.86	3.41
0.40	68.96	69.98	1.02	1.47

Table 12 Calculation values by Eq. (8) and measurement values of grounding resistance

$\mathrm{h} = 1\mathrm{m},\ S = 1 \times 1\mathrm{m}^2,\ \rho = 523\Omega \cdot m$

Thickness of reduction material d, m	Grounding resistance, Ω		deviation, Ω **ΔR**	deviation rate, % **ΔR**(%)
	R2	R0		
0.05	130.23	115.26	-14.97	12.98
0.20	130.23	87.95	-42.28	48.07
0.40	130.23	70.34	-59.89	85.14

Table 13 Calculation values by Eq. (13) and measurement values of grounding resistance

$\mathrm{t} = 0.02\mathrm{m}$ $\rho = 523\Omega \cdot m$

Resistivity of reduction material $\boldsymbol{\rho_c}$, **Ωm**	Grounding resistance, Ω		deviation, Ω **ΔR**	deviation rate, % **ΔR**(%)
	R1	R0		

8	121.07	123.97	2.9	2.39
20	121.28	124.76	3.48	2.86
40	121.64	124.95	3.31	2.72

Table 14 Calculation values by Eq. (8) and measurement values of grounding resistance

$h = 1\text{m},\ S = 1 \times 1\text{m}^2, \rho = 523\Omega \cdot m$

Resistivity of reduction material ρ_c, Ωm	Grounding resistance, Ω		deviation, Ω ΔR	deviation rate, % ΔR(%)
	R2	R0		
8	130.23	124.65	-5.58	4.47
20	130.23	124.89	-5.34	4.27
40	130.23	125.12	-5.11	4.08

IV. Discussion

It is noted that deviation rates between calculation values by Eq. (13) and analysis values by ANSIS program are less than 3% from Table 1-3. Also it is noted that deviation rates between calculation values by Eq. (13) and field measurement values are less than 5% from Table 4-9. These mean that Eq. (13) about grounding resistance of plate board electrode has high accuracy.

The range of soil resistivity used in field measurement examination is 500-900Ω·m. The accuracy of the grounding resistance of plate board electrode is improved by the mathematical model in this paper.

In case of using reduction material on the plate board electrode, variety factors have considered in the calculation of grounding resistance of the plate board electrode, thus the mathematical model is complex than before.

V. Conclusion

In this paper, a mathematical model on the grounding resistance of the plate board electrode considering the influence of embedding depth and the thickness and resistivity of reduction material in homogeneous soils, has been obtained. To verify the accuracy of the proposed calculation equation on the grounding resistance, the simulation and field measurement results were carried out the range of 500-900 Ω · m on the soil resistivity.

From the results of improved mathematical model Eq. (13) on the plate board electrode when compared with the previous and mathematical model Eq. (8) in case of using reduction material, it is concluded that it can be used effectively to the design of grounding system using reduction material.

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